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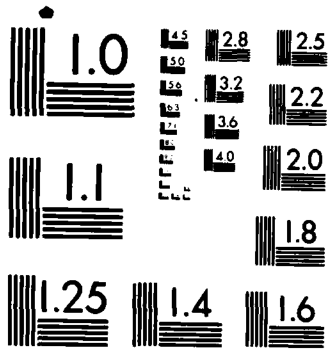

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FINAL TECHNICAL REPORT

NUMERICAL GENERATION OF 3D CURVILINEAR COORDINATE SYSTEMS AND COMPUTATIONAL GRIDS FOR AIRCRAFT CONFIGURATIONS

AD-A162 249

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by

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Abstract

This report describes the technical progress under a two-year research effort to investigate the properties of the Lockheed 3-D elliptic grid generation procedure and improvement of its capabilities for treating complex geometric features characteristic of aircraft configurations. ~~The initial~~ investigation identified two areas of weakness in the original technique. Both relate to the fundamentally important capability for generating curved-surface grids as a necessary prerequisite for generating 3-D space grids. The ~~areas of~~ weakness were: ~~(i)~~ (1) difficulties associated with the limitations of Cartesian coordinates for describing convex surfaces; and (ii) problems in controlling the orthogonality of the grid near boundaries.

The bulk of the first year's research ~~was~~ focussed on the first weakness, and resulted in development of a generalized system of elliptic grid generation equations that allow both the surface and the grid to be described in terms of an arbitrary system of curvilinear coordinates. Research in the second year was

The second year of the contract has been concerned primarily with the problem of grid orthogonality near boundaries. Three approaches ~~have been~~ investigated to improve the control that the user has over the angle at which grid lines intersect the boundaries. were; ~~The three approaches are~~ (1) refinement of the original techniques; (2) development of techniques for marching from a given boundary using the angle as an initial condition, and (3) derivation of a fourth-order system that permits additional boundary conditions on the grid intersection angle.

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Section 1
INTRODUCTION

This report describes the technical progress under a two-year research effort to investigate the properties of a 3-D elliptic grid generation procedure and improve its capabilities for treating complex geometric features characteristic of aircraft configurations. The grid generation technique that was the subject of the research was an outgrowth of earlier Lockheed work on two-dimensional grids for computing the flow in and about three-dimensional aircraft jet exhaust nozzles [Thomas (1979, 1980), Thomas and Middlecoff (1980)]. In that work, the aim was to build up a three-dimensional grid as a collection of two-dimensional grids in successive cross-sectional planes along the nozzle. A method was developed that was patterned after the earlier work of Thompson [Thompson,1974] in which a 2-D grid is generated numerically as the solution to an elliptic boundary value problem. We improved upon this approach by incorporating a set of general grid control functions into the differential operator of the elliptic system of partial differential equations. In this method, we evaluated the grid control functions at the boundaries and interpolated them into the interior of the region to provide direct control over the behavior of the interior grid by making it mimic the boundary shape and grid point distribution [Thomas, 1979; Thomas and Middlecoff, 1980]. Later, we extended this two-dimensional elliptic grid generation technique to three dimensions [Thomas, 1981]. The latter technique is referred to herein as the Lockheed 3-D elliptic grid generation procedure, and has been the subject of the two-year research effort described in this report.

During the first year of the effort, research was conducted to investigate the capability of the Lockheed 3-D elliptic grid generation procedure for handling complex geometrical features characteristic of aircraft configurations. The initial investigation identified two areas of weakness in the original technique. Both relate to the fundamentally important capability for generating curved-surface grids as a necessary prerequisite for generating 3-D space grids. The areas of weakness were: (i) difficulties associated with the limitations of Cartesian coordinates for describing convex surfaces, and (ii) problems in controlling the orthogonality of the grid near boundaries. The latter prolems are more difficult to deal with in the case of two-dimensional grids on curved surfaces than for planar two-dimensional grids.

The bulk of the first year's research was focussed on the first weakness, and resulted in development of a generalized system of elliptic grid generation equations that allow both the surface and the grid to be described in terms of an arbitrary system of curvilinear coordinates [Whitney and Thomas, 1983]. This overcame the inherent limitations of Cartesian coordinate representations. Some progress also was made in improving the orthogonality near the boundaries by the ad hoc method of specifying the curvature distribution that enters into the evaluation of the grid control functions.

As a result of the first year of effort, it was concluded that the method had good potential for applications. In fact, other related activities included application of the

method to practical cases, such as a 3D grid over a wing-fuselage combination, for use with the FLO-57 transonic flow code [Thomas and Neier, 1983], and development of software for interactive application of the technique.

The second year of the contract has been concerned primarily with the problem of grid orthogonality near boundaries. Three approaches have been investigated to improve the control that the user has over the angle at which grid lines intersect the boundaries. The three approaches are (1) refinement of the original techniques, (2) development of techniques for marching from a given boundary using the angle as an initial condition, and (3) derivation of a fourth-order system that permits additional boundary conditions on the grid intersection angle.

A study has been made to examine various ways of refining the original grid control technique to allow direct control over the angle with which the transverse grid lines meet the boundaries of the region in which the grid is generated. These are the boundaries at which the grid control parameters in the elliptic system are evaluated in terms of a pre-assigned distribution of boundary points. One effective refinement has been developed that is based on the fact that the equation defining each grid control parameter at a boundary is composed of two terms, a grid curvature term and a grid stretching term. In the refined technique, the two terms are evaluated on different parts of the boundary, are interpolated separately into the interior of the region, and are summed to form the grid control parameter. In contrast, the original technique evaluated both terms along the same part of the boundary, and the grid control parameter itself was interpolated into the interior. The refined technique improves considerably the user's control of the grid, but also has some limitations: (1) it does not change the local behavior of interior grid lines when the boundary segments are straight, (2) it ensures orthogonality of grid lines to a boundary only when the two boundary segments belonging to the same family as the grid lines are themselves both curved and orthogonal to the boundary in question, and (3) the technique is less useful for two-dimensional grids on curved surfaces than for either planar two-dimensional grids or three-dimensional space grids.

The marching approach is based on using our original second-order system of elliptic equations, including the grid control functions, and solving the equations by forward integration away from an initial boundary at which both the grid point locations and grid intersection angles are specified as initial conditions. This approach was abandoned when we were unable to develop a stable marching procedure for general grid topologies.

As discussed previously, the original second-order system cannot accommodate specified grid intersection angles at the boundaries. To overcome this shortcoming, one approach is to go to a higher-order system of partial differential equations which admit more boundary conditions. We have investigated such a fourth-order system obtained by applying the Laplacian differential operator to the original second-order system. This particular system has the advantage of retaining the grid control functions inherent in the original second-order system while allowing the grid intersection angles at boundaries to be specified as boundary conditions. The resulting system is significantly more complex than the second-order system. Numerical results have been obtained for several representative cases in two dimensions, but further effort would be required to develop a reliable and efficient algorithm for solving the fourth-order equations in three dimensions.

The remainder of this report contains a detailed discussion of the described research. The objectives of each year of the effort are given in Section 2 as they appeared in the original contract statements of work. The research performed toward those objectives is discussed in Section 3. Finally, Section 4 lists the technical papers prepared under the contract and the personnel who participated in the research.

Section 2

OBJECTIVES OF THE RESEARCH

The research objectives of each year of the research effort are given below as they appeared in the contractual statements of work.

2.1 Objectives of the First Year of Effort

Investigate the capability of the Lockheed 3-D elliptic grid generation procedure in handling complex geometrical features which are characteristic of aircraft configurations. Determine limitations in the procedure by considering model problems which include specific geometric features which may violate known or suspected restrictions in the method. Eliminate current constraints which exclude non-orthogonally intersecting surfaces, such as the intersection of a swept wing with a fuselage.

2.2 Objectives of the Second Year of Effort

Investigate ways to control the angle at which grid lines intersect boundaries. Three approaches will be considered: (1) refinement of grid control techniques on the current method, (2) development of march solution techniques that would allow the angle of intersection with the boundary to be prescribed as an initial condition, and (3) derivation of a fourth-order elliptic system based on the present second-order system to provide additional boundary conditions on the grid angle at the surface.

Section 3

SUMMARY OF THE RESEARCH ACCOMPLISHED

The following paragraphs summarize the research performed in each year of the effort toward the objectives stated in Section 2.

3.1 Research Performed During the First Year of Effort

The effort during the first year was focussed primarily on the use of elliptic equation systems to generate quasi-two-dimensional grids on curved surfaces typical of the surfaces found on aircraft configurations. The ability to generate smooth, well-controlled surface grids is a necessary prerequisite for three-dimensional grid generation by elliptic differential systems, because the surface grid on the boundary of a 3-D region forms the boundary values that are needed to solve the 3-D elliptic equation system that generates the 3-D grid.

Earlier, we had developed a special elliptic equation system for generating grids on surfaces $z=f(x,y)$ in Cartesian coordinates [Thomas,1981]. This system is similar to the one we had used previously for plane 2-D grids [Thomas and Middlecoff (1979)], but contains forcing function "source terms" that involve the slope and curvature of the surface. During the first year, two difficulties were discovered in the use of this Cartesian-based elliptic system: (i) problems in obtaining a converged solution for the grid on fuselage-like or wing-like surfaces in regions where the derivatives of the surface shape function $f(x,y)$ are large or are rapidly-varying, and (ii) difficulties with the local control of grid lines near boundaries, where near-orthogonality of the grid lines to the boundary usually is desirable, and often is absolutely necessary to assure smoothness of the grid across boundaries between different regions.

The first problem is a consequence of using Cartesian coordinates in terms of which the surface shape function $f(x,y)$ is always multiple-valued and has regions of locally unbounded derivatives when used to describe a convex geometrical object such as an aircraft or its components (fuselage, wings, nacelles, etc). It became apparent to us that this problem could be avoided by using other coordinate systems, such as cylindrical, spherical, etc., that are both more natural and more convenient.

The major part of the first year's effort, therefore, was applied to develop a generalized system of elliptic surface grid generation equations that allows one to describe the surface and generate the grid in terms of any arbitrary curvilinear coordinate system. Our earlier Cartesian-based surface grid generation system is simply a special case of the generalized system. The results of this work were reported in a technical paper at the Mini-Symposium on Advances in Grid Generation [Whitney and Thomas, 1983].

The second problem mentioned above, namely grid control, also was addressed initially during the first year. It was found that our original techniques of controlling the behavior of the grid through the boundary values for the elliptic system are much less powerful for dealing with grids on curved surfaces than we had found them to be for plane grids. An example is given in Fig. 1 to demonstrate the inability of the original grid control technique to produce grid lines that are orthogonal to the boundaries. The figure shows a grid on the surface of a bilaterally symmetric fuselage, a spherically-capped cylinder with a

cutout for a mid-wing having a NACA 0012 profile. The grid was generated by the elliptic equation system mentioned earlier, using the grid control techniques originally developed for plane grids. One can see that the grid lines are quite skewed and do not intersect the longitudinal boundaries orthogonally except very near the nose tip and again very near the aft end. Clearly, if this grid for the upper half of the fuselage were joined to a similar one for the lower half, the "vertical" grid lines would have a sharp discontinuity in slope as well as in curvature in crossing the boundary where the two halves join. A similar skewness occurs along the boundary between the right and left halves, the former of which is not shown.

Some progress was made during the first year to improve the orthogonality near the boundaries by specifying the curvature distribution that enters into one of the two terms that are used in evaluating the grid control functions at each boundary. The grid shown in Fig. 2 was produced in this way, and has transverse grid lines that are very nearly orthogonal to all boundaries, including those where the original grid of Fig. 1 is markedly skewed.

3.2 Research Performed During the Second Year of Effort

Three different approaches were investigated during the second year to improve control over the angle with which the grid lines intersect boundaries: refinement of the existing grid control technique, marching methods that would allow the angle of intersection with an initial boundary to be prescribed as an initial condition, and manipulation of the original second-order system to derive a fourth-order elliptic system that would permit additional boundary conditions on the intersection angle at all boundaries. Progress in each of these areas is outlined below.

3.2.1 Refinement of the Original Technique

We have studied various ways of refining the original grid control technique to improve control over the angle with which the transverse grid lines meet the boundaries of the region in which the grid is generated. These are the boundaries at which the grid control parameters in the elliptic system are evaluated in terms of a pre-assigned boundary grid point distribution. To simplify the discussion, we shall use a two-dimensional example to contrast the original technique with the newly-developed refinements.

Overview of the Original Technique. Given a simply-connected region of the x,y plane such as the region illustrated in Fig. 3, the original technique generates a boundary-conforming curvilinear coordinate transformation

$$x, y \longmapsto \xi, \eta$$

$$\xi = \xi(x, y), \qquad \eta = \eta(x, y)$$

that maps the region onto a rectangle (Fig. 4). The transformation is generated numerically as the solution to the elliptic system of equations

$$\nabla^2 \xi = \phi(\xi, \eta)\,|\nabla \xi|^2, \qquad \nabla^2 \eta = \psi(\xi, \eta)\,|\nabla \eta|^2 \tag{1}$$

where ϕ, ψ are grid control functions. The roles of dependent and independent variables in these equations are interchanged to obtain an elliptic system that can be solved numerically on a uniform rectangular grid in the domain of Fig. 4 to obtain the grid point coordinates in the physical region

$$\alpha(\vec{r}_{\xi\xi} + \phi\vec{r}_{\xi}) - 2\beta\vec{r}_{\xi\eta} + \gamma(\vec{r}_{\eta\eta} + \psi\vec{r}_{\eta}) = 0 \tag{2}$$

$$\vec{r} = (x, y) \tag{3a}$$

$$\alpha = |\vec{r}_{\eta}|^2, \qquad \beta = (\vec{r}_{\xi} \cdot \vec{r}_{\eta}), \qquad \gamma = |\vec{r}_{\xi}|^2 \tag{3b}$$

Boundary values $\vec{r}_{jk}$ are assigned along the perimeter O'A'B'C'O' of the square. These boundary values represent the coordinates $\vec{r} = (x, y)$ of grid points along the perimeter OABCO of the physical region.

In the original grid control technique, limiting forms of Eq. (2) were used to evaluate the control functions ϕ, ψ locally along the boundary of the square, so that the elliptic system then could be solved numerically. The functions were evaluated individually along each of the four segments of the boundary as follows.

Consider, for example, the lower horizontal segment O'A', which represents the boundary OA in Fig. 3, and is a coordinate curve $\eta = constant$ along which ξ alone varies. A unique equation for evaluating ϕ on this segment follows by taking the projection of Eq. (2) onto the vector $\vec{r}_{\xi}$ that is tangent to the segment, and invoking the orthogonality constraint

$$\vec{r}_{\xi} \cdot \vec{r}_{\eta} = 0 \qquad on \quad O'A'B'C'O' \tag{4}$$

The resulting equation can be cast in the form

$$\phi = -(T_0 + |\vec{r}_{\xi}| T_1) \tag{5a}$$

$$T_0 = (\vec{r}_{\xi} \cdot \vec{r}_{\xi\xi})/|\vec{r}_{\xi}|^2 = (|\vec{r}_{\xi}|^2)_{\xi}/(2|\vec{r}_{\xi}|^2) = (ln|\vec{r}_{\xi}|)_{\xi} \tag{5b}$$

$$T_1 = (\vec{r}_{\xi} \cdot \vec{r}_{\eta\eta})/(|\vec{r}_{\xi}||\vec{r}_{\eta}|^2) \tag{5c}$$

The term T_0 is simply the logarithmic derivative of arc length with respect to the parameter ξ along the boundary segment [Thomas, 1981]. It depends on the spacing between grid points along the boundary segment OA, and will be referred to as the "grid stretching" term. Both of the ξ-differentiated quantities $|\vec{r}_{\xi}|$ and T_0 in Eqs. (5) can be evaluated numerically from the boundary values assigned on the segment O'A'. However, the term T_1 involves η-derivatives which cannot be evaluated locally using only boundary

data on O'A'. This term represents the local curvature [Thomas, 1981] of the grid lines $\xi = constant$ that are transverse to the boundary segment OA and will be referred to as the "grid curvature" term.

Clearly, one could specify any desired curvature distribution function $T_1(\xi)$ along the boundary segment. We shall return to this idea later. In the original grid control technique, however, this curvature term was simply evaluated at the end points of the segment OA, and was given a linear variation over the rest of the segment for use in evaluating the grid control function ϕ from Eq. (5a). The same procedure was employed to evaluate the parameter $\phi(\xi)$ along the upper boundary segment B'C' in Fig. 4. A continuous representation $\phi(\xi, \eta)$ throughout the interior of the computational square was then obtained by linear interpolation as a function of η along vertical mesh lines $\xi = constant$.

Outline of The Refined Technique. Refinements have been added to the original technique to improve the grid control both in the interior of the region and near the boundaries when the boundaries are curved. The refinements are based on a new approach that makes direct use of the physical interpretation of the term T_1 in Eq. (5b) as the curvature of η-directed grid lines. The rationale is as follows.

The purpose of the grid control functions ϕ, ψ is to make the elliptic system (2) produce an interior grid that mimics the shape and curvature of the boundaries and the grid point distribution that is assigned on those boundaries. Consider a point P' at which the grid control function is to be evaluated (see Fig. 4), and let P be the image of that point in the physical region of Fig. 3. The horizontal and vertical grid lines in Fig. 4 that pass through the point P' have as their image in the physical region two intersecting grid lines that are illustrated by the dashed lines in Fig. 3. If these two physical grid lines were known explicitly, then one could evaluate the function ϕ locally from Eq. (5). The first term could be calculated if one knew only the ξ-directed grid line through P; this is the grid stretching term that reflects the grid spacing along the ξ-directed grid line. The second term represents the curvature of the η-directed grid line through P, but requires knowledge of both grid lines since it contains derivatives with respect to both ξ and η. However, one can make use of the orthogonality constraint (4) to rewrite Eq. (5c) in terms of η derivatives alone, as

$$T_1 = \left(\frac{\vec{r}_\eta}{|\vec{r}_\eta|} \times \vec{k}\right) \cdot \frac{\vec{r}_{\eta\eta}}{|\vec{r}_\eta|^2} \tag{6}$$

where $\vec{r} = (x, y, z)$ with $z = 0$ and $k = (0, 0, 1)$ is the Cartesian unit vector in the z-direction normal to the x, y plane. Eq. (6) follows from the fact that the orthogonality relation (4) can be written in the form

$$\frac{\vec{r}_\xi}{|\vec{r}_\xi|} = \frac{\vec{r}_\eta}{|\vec{r}_\eta|} \times \vec{k} \tag{7}$$

The foregoing conceptual exercise suggests a rational way to compute the grid control function ϕ when the grid is known only on the boundaries, and one would like the interior

grid produced by the elliptic system to be some smooth interpolation from the known grid on the boundaries: the grid-stretching term T_0 governs the local grid spacing along the ξ-directed grid line through the point P, and should be estimated in terms of the known grid spacing on the ξ-directed boundary segments OA and CB. The curvature term T_1 governs the local curvature of the η-directed grid line through P and should be estimated in terms of the known curvature of the η-directed boundary segments OC and AB.

The refined grid control procedure then is as follows. We compute the grid control function ϕ at a general interior point P' of the square in Fig. 4 by using Eq. (5a) locally. The grid stretching term T_0 is computed at the boundaries O'A' and C'B' and interpolated linearly as a function of η into the interior of the square. The factor $|\vec{r}_\xi|$ in Eq. (5a) is evaluated in the same way. The curvature term is computed at the boundaries OC and AB by using Eq. (6), and is interpolated linearly as a function of ξ into the interior of the square.

The described refinement greatly improves both the behavior of the interior grid and the orthogonality of grid lines at the boundaries whenever the boundary segments are curved and are nearly orthogonal to one another. The improvement is illustrated by the example shown in Fig. 5 for a region that has two curved boundary segments that are nearly orthogonal to the horizontal straight segment. One sees from Fig. 5a that the original technique produces interior grid lines that roughly follow the shapes of the two curved boundary segments, but that have too little curvature and do not mimic the orthogonality with which the curved boundary segments meet the horizontal straight segment. In contrast, the curved interior grid lines produced by the refined grid control technique follow faithfully the curvature of the boundary segments, and also are nearly orthogonal to the horizontal straight segment. This has the added effect of making the spacing between adjacent interior curved grid lines follow closely the spacing of the assigned grid points along the straight boundary segments.

The refined grid control technique in general gives similar improvements both in two-dimensional plane grids and in three- dimensional space grids whenever the boundaries are curved. However, there are some limitations.

1. The refinements do not change the behavior of interior grid lines when the boundary segments are straight. For example, there is little difference between Figs. 5a and 5b in the degree of orthogonality with which the "radial" interior grid lines meet the upper (right-hand) curved boundary, because the "radial" boundary segments (the horizontal and vertical boundary segments) have no curvature.

2. The refined technique cannot produce a family of interior grid lines that are orthogonal to a boundary unless the two boundary segments in that family are both curved and orthogonal to the boundary in question.

3. The refined technique is useful (within the foregoing two limitations) for either planar two-dimensional grids or three-dimensional space grids, but offers less generality in improving two-dimensional curved-surface grids generated by the 2-D elliptic systems developed the first year of the contract [Whitney and Thomas, 1983].

The loss of generality in the case of 2-D curved surface grids stems from the fact that the curvature term T_1 in Eq.(5c) does not embody the actual principal curvature of the space curve that bounds a general 2-D curved surface region as it does for the planar

surface. T_1 actually represents only the component of the curvature vector that lies in the tangent plane to the surface, and is generally equal to the principal curvature only in the planar case. This means that the effectiveness of the grid control functions ϕ, ψ depends on the detailed nature of the curved surface and on the shape of the space curve that bounds the part of the surface on which a grid is to be generated.

As discussed in Section 3.1, we have been able to enhance the grid control for some specific cases of surface geometries and grid topologies by specifying a priori the curvature distribution such as $T_1(\xi)$ along the $\eta = constant$ lines in the computational domain of Fig. 4. We have found that this introduction of ad hoc curvature control can be applied on a trial and error basis to force the grid to be nearly orthogonal at boundaries, at least for relatively simple surface geometries and grid topologies. However, the ad hoc approach lacks the generality of the marching and fourth-order system approaches discussed below.

3.2.2 Marching Approach

The marching approach is closely related with the use of parabolic equations to construct the grid by solving an initial value problem [Nakamura (1982)]. The parabolic marching idea is being pursued by other investigators, and is largely concerned with devising a useful system of equations and with learning how to control the behavior of the grid everywhere, not just near boundaries. In contrast, the marching approach investigated under the contract was based on our original second-order system of elliptic equations, including the grid control functions that are evaluated at all boundaries. Because the grid generation equations are of second order, one could, in principle, specify two boundary conditions at the initial boundary from which the solution is to be marched: one boundary condition on the grid point locations along the boundary, and a second on the angle at which the grid lines intersect the boundary. This would allow complete control over the grid behavior at the initial boundary by sacrificing a degree of control over the behavior at the far boundary toward which the solution is marched.

Marching solutions are rarely attempted for elliptic equations because the initial-boundary value problem is ill-posed and numerical stability is difficult to maintain. In our original elliptic system, however, global information about the boundary values is contained in the grid control functions that appear in the equations themselves. It seemed reasonable to hypothesize that this global boundary information content might alter the character of the equations enough to allow stable solutions by marching away from an initial boundary. Unfortunately, this hypothesis did not prove true in practice. Numerical experiments were conducted in which a locally linearized implicit marching technique was attempted for planar 2-D grids. We were unable to obtain stable marching solutions for general grid topologies, and abandoned the marching approach in favor of the fourth order system approach discussed next.

3.2.3 Fourth-Order System Approach

The idea of using a fourth-order elliptic grid generation system that allows the freedom to specify boundary conditions on the grid point locations and on the angle of intersection of grid lines with the boundary has been examined to a limited extent [Shubin, et al. (1982)]. That study considered only the use of the biharmonic equation, a linear equation

that satisfies no maximum principle. The investigators noted that unacceptable singular transformations with grid lines that cross others of the same family were encountered even in relatively simply-shaped regions. It is likely that this difficulty would be much more probable for more complex geometries typical of aircraft, and would be much harder to overcome.

Our original second-order nonlinear elliptic systems obey a maximum principle that ensures a non-singular grid. We have investigated ways to develop a fourth-order system with this property by applying a further second-order linear differential operator to the existing second-order nonlinear elliptic system. It is plausible to conjecture that the existence of a maximum principle for the present basic second-order system should imply that a fourth-order system derived from it also would obey the same maximum principle. Such a fourth-order system would retain the grid control functions and therefore the quality of interior grid control that is obtained with the second-order system, and would also allow complete local control over the angle at which grid lines intersect the boundary.

The system that immediately suggests itself is obtained by operating on the original second-order system with the Laplacian operator; i.e.,

$$\nabla^2\{\nabla^2\xi_i - \phi_i\,|\nabla\xi_i|^2\} = 0, \qquad i = 1,2,3 \tag{8}$$

where $(\xi_1, \xi_2, \xi_3) = (\xi, \eta, \varsigma)$ are the transformed coordinates, and $(\phi_1, \phi_2, \phi_3) = (\phi, \psi, \omega)$ are the grid control parameters. Just as for the second-order system, the transformed system of equations is derived by interchanging the roles of dependent and independent variables. Rather than working with a single fourth-order system for each grid variable, it is convenient to consider the following coupled second-order system:

$$\nabla^2\xi_i = \phi_i\,|\nabla\xi_i|^2 + \rho_i, \qquad \nabla^2\rho_i = 0, \qquad i = 1,2,3 \tag{9}$$

The functions ρ_i are dummy variables that serve to reduce the order of the original system in Eq. (8). It is easily verified that Eq. (8) is obthained by eliminating ρ_i from Eq. (9). The system in Eq. (9) transforms to the quasi-linear system

$$\sum_i \left\{ C_{ii}\left(\frac{\partial^2\vec{r}}{\partial\xi_i^2} + \phi_i\frac{\partial\vec{r}}{\partial\xi_i}\right) + \rho_i J^2 \frac{\partial\vec{r}}{\partial\xi_i} \right\} + 2\sum_{\substack{i,j\\ i<j}} C_{ij}\frac{\partial^2\vec{r}}{\partial\xi_i\partial\xi_j} = 0 \tag{10}$$

$$\sum_i \left\{ C_{ii}\left(\frac{\partial^2\vec{\rho}}{\partial\xi_i^2} + \phi_i\frac{\partial\vec{\rho}}{\partial\xi_i}\right) + \rho_i J^2 \frac{\partial\vec{\rho}}{\partial\xi_i} \right\} + 2\sum_{\substack{i,j\\ i<j}} C_{ij}\frac{\partial^2\vec{\rho}}{\partial\xi_i\partial\xi_j} = 0 \tag{11}$$

in which the coefficients C_{ij}, and the Jacobian J are given by

$$C_{li} = \left(\frac{\partial\vec{r}}{\partial\xi_m}\cdot\frac{\partial\vec{r}}{\partial\xi_j}\right)\left(\frac{\partial\vec{r}}{\partial\xi_n}\cdot\frac{\partial\vec{r}}{\partial\xi_k}\right) - \left(\frac{\partial\vec{r}}{\partial\xi_m}\cdot\frac{\partial\vec{r}}{\partial\xi_k}\right)\left(\frac{\partial\vec{r}}{\partial\xi_n}\cdot\frac{\partial\vec{r}}{\partial\xi_j}\right)$$

$$J = \frac{\partial\vec{r}}{\partial\xi_1}\cdot\left(\frac{\partial\vec{r}}{\partial\xi_2}\times\frac{\partial\vec{r}}{\partial\xi_3}\right)$$

In the definition of C_{li} , the indices (i, j, k) and (l, m, n) occur in cyclic order. These equations hold for each physical-space coordinate, $\vec{r} = (x, y, z)$, and for each "dummy variable", $\vec{\rho} = (\rho_1, \rho_2, \rho_3)$. There are, therefore, a total of six scalar equations. These equations bear a close resemblance to the original equation system, which may be written as

$$\sum_i C_{ii}\Big(\frac{\partial^2 \vec{r}}{\partial \xi_i^2} + \phi_i \frac{\partial \vec{r}}{\partial \xi_i}\Big) + 2 \sum_{\substack{i,j \\ i<j}} C_{ij} \frac{\partial^2 \vec{r}}{\partial \xi_i \partial \xi_j} = 0 \tag{12}$$

The only difference is the occurrence of the J^2 term in Eqs. (10) and (11). With slight modification, the same SLOR solution technique used on the oritinal system (12) may be employed here, only now there are are six second-order equations instead of three.

It remains to determine appropriate boundary conditions. Boundary conditions for $\vec{r}$ are the same as for the original second order system (12); i.e., $\vec{r}$ is specified at each boundary node. Grid intersection slopes on the boundaries may be specified by also giving the normal derivative of $\vec{r}$ at the boundary. However, since the system Eqs. (10) and (11) is second order, only boundary conditions for $\vec{r}$ and $\vec{\rho}$ need be given. A Dirichlet boundary condition for each component of $\vec{\rho}$ can be found from the first equation in Eq. (9), which transforms to Eq. (10). The boundary condition is applied in the context of an SLOR iteration loop, for which it is assumed that an approximate, or guessed, value of $\vec{r}$ *is known at each node for a particular iteration step.* The boundary condition is obtained from the limiting form of Eq. (10) at the boundaries. For example, consider the boundary condition on $\xi_3 = \varsigma = 1$. On this surface suppose that $\vec{r}$ and $\vec{r}_\varsigma$ are given input data. Then by differentiation, all the other first and second partial derivatives of $\vec{r}$ are known except for $\vec{r}_{\varsigma\varsigma}$. But this can be estimated from $\vec{r}$ and $\vec{r}_\varsigma$ at the boundary plus $\vec{r}$ on the first layer of nodes interior to the boundary; i.e.,

$$\frac{\partial^2 \vec{r}}{\partial \varsigma^2}\Big]_{\varsigma=1} \simeq \frac{2}{h^2}\Big(h \frac{\partial \vec{r}}{\partial \varsigma}\Big]_{\varsigma=1} - \vec{r}\,\Big]_{\varsigma=1} + \vec{r}\,\Big]_{\varsigma=1-h}\Big) \tag{13}$$

With $\vec{r}_{\varsigma\varsigma}$ so approximated, and similar expressions on the other boundaries, the three scalar equations represented by Eq. (10) may be used to solve for the three ρ_i's at the boundaries. Interior values of $\vec{\rho}$ are found by solving Eq. (11).

A numerical solution to the fourth-order system of equations has been obtained for several cases. The numerical method employs finite-difference approximants of Eqs. (10) and (11) that incorporate Dirichlet boundary conditions for $\vec{r}$ and $\vec{\rho}$. The solution then is obtained by an iterative technique based upon the successive line over-relaxation (SLOR) algorithm. Numerical results have been obtained for several two-dimensional cases, but further effort would be required to develop a reliable algorithm for solving the fourth-order system in three dimensions. The two-dimensional numerical results are given in [Whitney, 1985], along with a complete derivation of the fourth-order system and a detailed description of the iterative solution technique.

Section 4

LIST OF TECHNICAL PAPERS AND PARTICIPATING PERSONNEL

The following subsections list the technical papers that have been prepared under the contract, and the personnel who participated in the research.

4.1 Technical Papers

The technical papers and reports prepared under the contract are listed below.

Whitney, A. K., and Thomas, P.D., *Construction of Grids on Curved surfaces Described by Generalized Coordinates through the use of an Elliptic System,* **Proceedings of the Mini-Symposium on Advances in Grid Generation**, sponsored by the Americal Society of Mech. Engineers, Houston, Texas, June, 1983.

Thomas, P.D., *Stationary Interior Grids and Computation of Moving Interfaces*, **Advances in Computational Methods for Boundary and Interior Layers**, Edited by J. J. H. Miller, Boole Press Ltd., Trinity College, Dublin, Ireland, pp. 82-87 1984.

Whitney, A. K., *A Fourth-Order Elliptic System for Grid Generation*, LMSC-F035605, Lockheed Missiles and Space Co., Inc., Research and Development Division,Palo Alto, Ca., March, 1985.

4.2 Participating Personnel

The following personnel participated in the research reported here.

K. L. Neier, B.S., Mathematics, Bucknell, 1966

R. R. Roloff, Ph.D., Computer Science, University of Illinois, 1979

P. D. Thomas (Principal Investigator), M.S., Engineering, UCLA,1958

A. K. Whitney, Ph.D., Engineering Science, Calif. Inst. of Technology, 1969

Section 5

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Nakamura, S., *Marching Grid Generation Using Parabolic Partial Differential Equations,* in **Numerical Grid Generation**, J.F. Thompson, Ed. Elsevier Science Publishing Co., Inc., pp. 667-686 , 1982.

Shubin, G.R., Stephens, A.B., and Bell, J.B., *Three Dimensional Grid Generation Using Biharmonics,* in **Numerical Grid Generation**, J.F. Thompson, Ed. Elsevier Science Publishing Co., Inc., pp. 667-686 , 1982.

Thomas, P. D., *Numerical Method for Predicting Flow Characteristics and Performance of Nonaxisymmetric Nozzles - Theory* NASA CR-3147, Sept. 1979.

Thomas, P. D., *Numerical Method for Predicting Flow Characteristics and Performance of Nonaxisymmetric Nozzles. Part 2 - Applications,* NASA CR-3264, October 1980.

Thomas, P. D. and Middlecoff, J. F., *Direct Control of the Grid Point Distribution in Meshes Generated by Partial Differential Equations,* AIAA Journal Vol. 18, No. 6, July 1980.

Thomas, P. D. *Construction of Composite Three-Dimensional Grids from Subregion Grids Generated by Elliptic Systems*, Paper No. 81-0996, **Proceedings of the AIAA 5th Computational Fluid Dynamics Conference**, Palo Alto, California, June 1981; AIAA Journal, Vol. 20, No. 9.

Thomas, P. D. *Numerical Generation of Composite Three- Dimensional Grids by Quasilinear Elliptic Systems,* in **Numerical Grid Generation**, J.F. Thompson, Ed. Elsevier Science Publishing Co., Inc., pp. 667-686 , 1982.

Thomas, P. D. and Neier, K. L., *Generation of Three- Dimensional Wing-Body Grids for the FLO-57 Code,* First Lockheed Corporate Symposium on Computational Aerodynamics, Burbank, California, April 1983.

Thomas, P.D., *Stationary Interior Grids and Computation of Moving Interfaces,* **Advances in Computational Methods for Boundary and Interior Layers**, Edited by J. J. H. Miller, Boole Press Ltd., Trinity College, Dublin, Ireland, pp. 82-87 1984.

Thompson, J.F., Thames, F.C., and Mastin, C.W., *Automatic Numerical Generation of Body-Fitted Curvilinear Coordinate System for Field Containing any Number of Arbitrary Two-Dimensional Bodies,* J. Comp. Phys., Vol. 15, p. 299, 1974.

Whitney, A. K., and Thomas, P.D., *Construction of Grids on Curved surfaces Described by Generalized Coordinates through the use of an Elliptic System,* **Proceedings of the Mini-Symposium on Advances in Grid Generation**, sponsored by the Americal Society of Mech. Engineers, Houston, Texas, June, 1983.

Whitney, A. K., *A Fourth-Order Elliptic System for Grid Generation,* LMSC-F035605, Lockheed Missiles and Space Co., Inc., Research and Development Division, Palo Alto, Ca., March, 1985.

Fig. 1 Fuselage Surface Grid Generated Using Standard Grid Controls

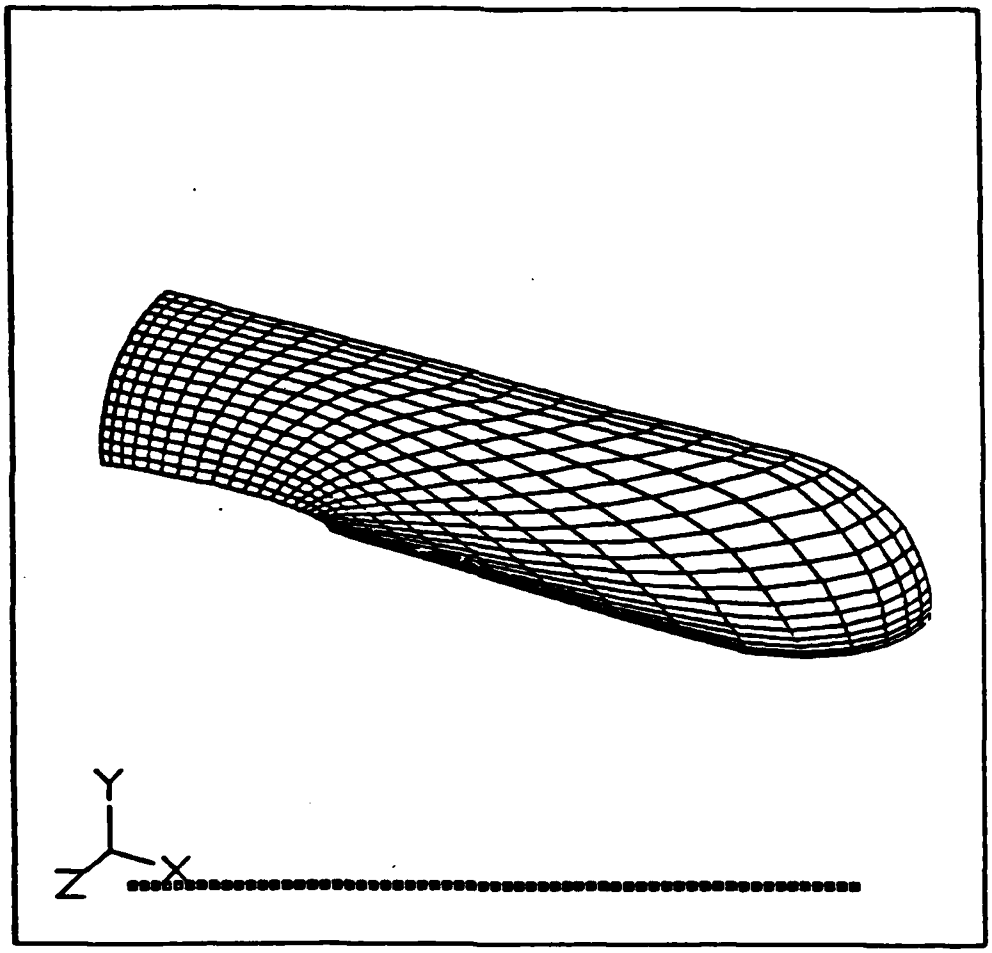
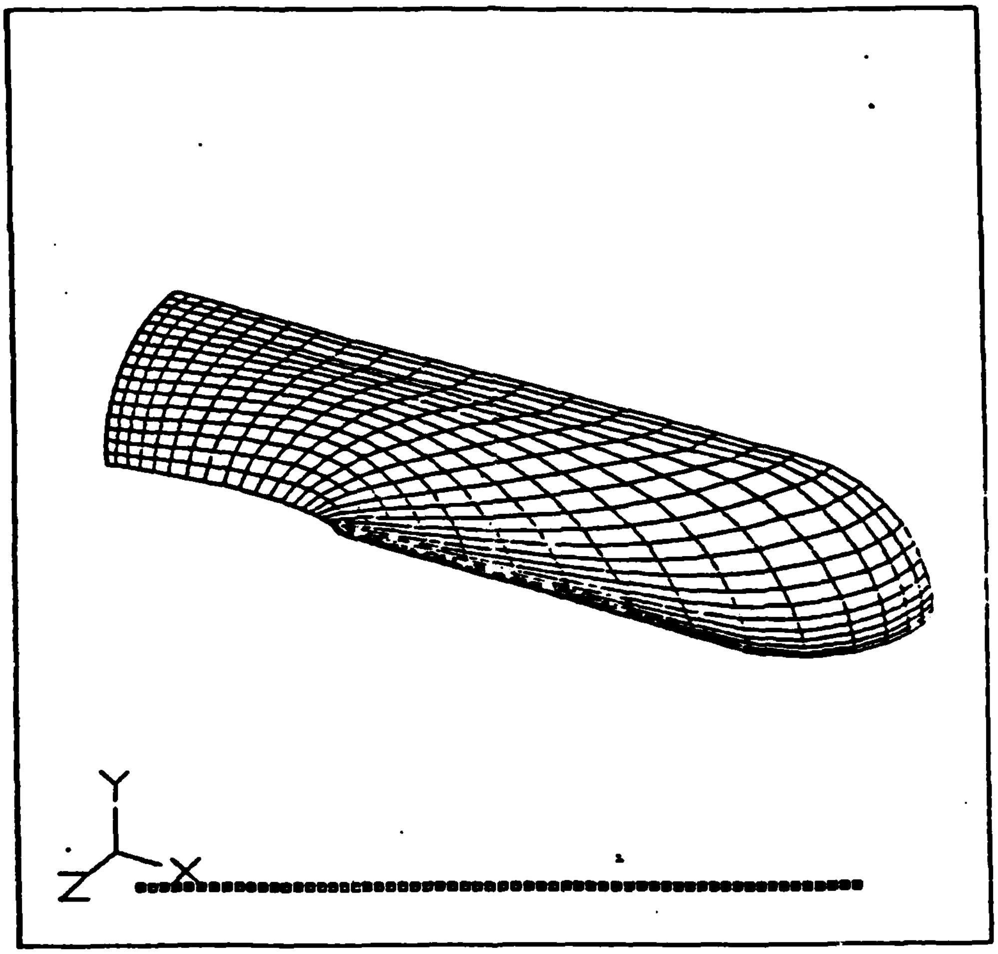


Fig. 2 Fuselage Grid Generated with Additional Curvature Controls



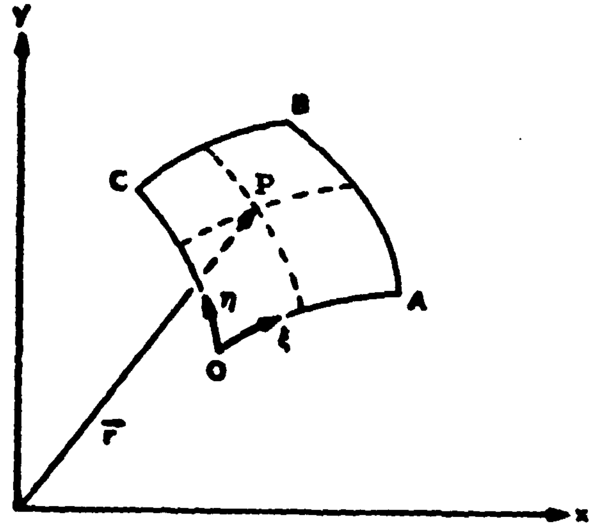


Figure 3 Two-dimensional Physical Region

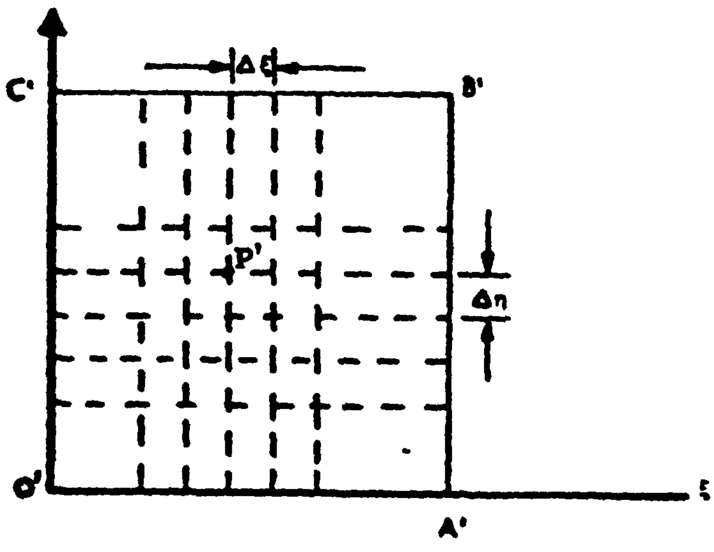
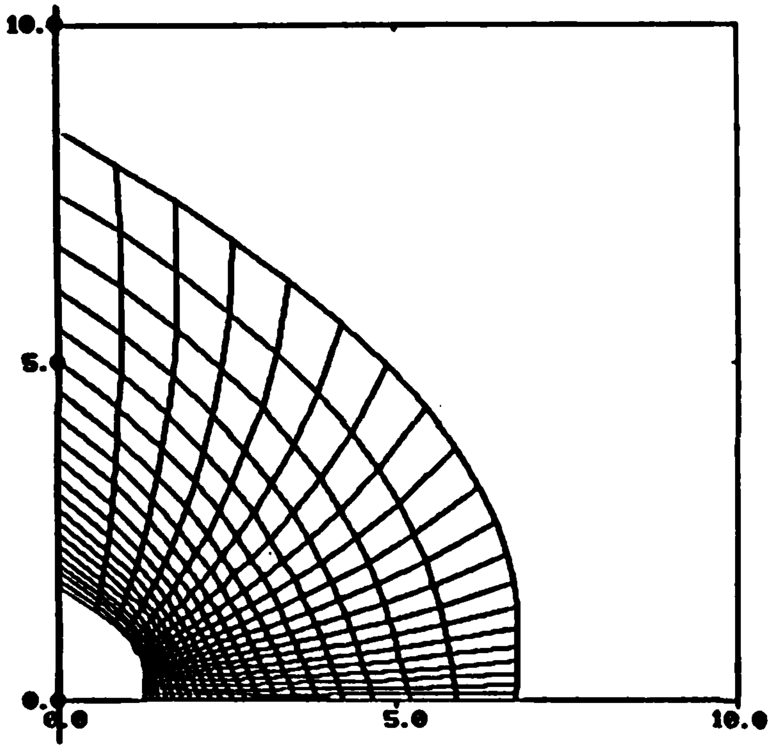
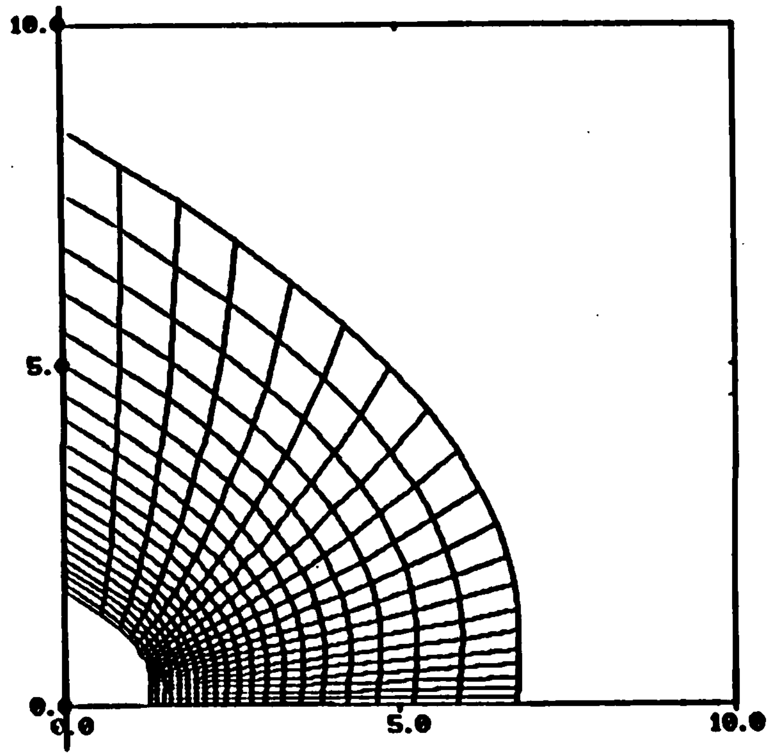


Figure 4 Rectangular Computational Domain



(a) Original technique.



(b) New refined technique.

Fig. 5 Comparison of the original and the new refined grid control techniques for a planar region with curved boundaries.

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This report describes the technical progress under a two-year research effort to investigate the properties of the Lockheed 3-D elliptic grid generation procedure and improve its capabilities for treating complex geometric features characteristic of aircraft configurations. The initial investigation identified two areas of weakness in the original technique. Both relate to the fundamentally important capability for generating curved-surface grids as a necessary prerequisite for generating 3-D space grids. The areas of weakness were: (i) difficulties associated with the limitations of Cartesian coordinates for describing convex surfaces, and (ii) problems in controlling the orthogonality of the grid near boundaries.

The bulk of the first year's research was focussed on the first weakness, and resulted in development of a generalized system of elliptic grid generation equations that allow both the surface and the grid to be described in terms of an arbitrary system of curvilinear coordinates.

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The second year of the contract has been concerned primarily with the problem of grid orthogonality near boundaries. Three approaches have been investigated to improve the control that the user has over the angle at which grid lines intersect the boundaries. The three approaches are (1) refinement of the original techniques, (2) development of techniques for marching from a given boundary using the angle as an initial condition, and (3) derivation of a fourth-order system that permits additional boundary conditions on the grid intersection angle.